MATH 147 Review: Partial Derivatives

Facts to Know

Notation

\[ f_x(x, y) = \frac{\partial f}{\partial x}, \quad f_y(x, y) = \frac{\partial f}{\partial y} \]

Rule for finding partial derivatives of \( z = f(x, y) \)

\[ f_x \quad x \text{ is the only variable (y is a constant)} \]

\[ f_y \quad y \text{ is the only variable (x is constant)} \]

Examples

1. Let \( f(x, y) = x^3 + x^2 y^3 - 2y^2 \). Find \( f_x \) and \( f_y \) and \( f_{xy} \) and \( f_{yx} \).

\[ f_x = \frac{\partial}{\partial x} (x^3 + x^2 y^3 - 2y^2) = \frac{\partial}{\partial x} (x^3) + \frac{\partial}{\partial x} (x^2 y^3) + \frac{\partial}{\partial x} (-2y^2) \]

\[ = 3x^2 + y^3 (2x) + 0 \]

\[ = 3x^2 + 2xy^3 \]

\[ f_{xy} = (f_x)_y = (3x^2 + 2xy^3)_y = 0 + 2x(3y^2) \]

\[ f_y = \frac{\partial}{\partial y} (x^3 + x^2 y^3 - 2y^2) = 0 + x^2(3y^2) - 4y \]

\[ f_{yx} = (f_y)_x = (x^2y^2 - 4y)_x = 6xy^2 \]
2. Let \( f(x, y) = 4 - x^2 - 2y^2 \). Find \( f_x(1, 1) \) and \( f_y(1, 1) \).

\[
\begin{align*}
\frac{\partial f}{\partial x} &= 0 + (-2x) + 0 \\
\frac{\partial f}{\partial y} &= 0 + 0 + (-4y)
\end{align*}
\]

\[
\begin{align*}
f_x(1,1) &= -2(1) = -2 \\
f_y(1,1) &= -4(1) = -4
\end{align*}
\]